Chapter 4

Assignment 4 Solutions

3.11. Consider a fused silica single-mode fiber that is 1 km long. Find the material dispersion component of the pulse spread $\Delta\tau$ if $\Delta\lambda = 3.0$ nm and the operating wavelengths are 800 nm, 900 nm, 1.3 μm and 1.5 μm.

Solution: The values of $\lambda^2 d^2n/d\lambda^2$ are estimated from Fig. 3.7b on p. 62 as shown below.

Wavelength (nm)	$\lambda^2\, d^2n/d\lambda^2$
800	0.026
900	0.017
1300	−0.001
1500	−0.008

The material dispersion is given by

$$\Delta\tau_m = -\frac{L}{c}\frac{\Delta\lambda}{\lambda}\lambda^2\left(\frac{d^2n}{d\lambda^2}\right) = -\left(\frac{1000}{3.0\times10^8}\right)\left(\frac{3\times10^{-9}}{\lambda}\right)\left(\lambda^2\frac{d^2n}{d\lambda^2}\right). \tag{4.1}$$

See the following table for the numerical solutions.

Wavelength (nm)	$\Delta\tau_m$ (seconds)
800	-3.25×10^{-10}
900	-1.889×10^{-10}
1300	$+7.69\times10^{-12}$
1500	$+5.16\times10^{-11}$

3.13. Consider a 0.80 km long fiber made of fused silica with a step-index. The following applies: $\lambda = 1.0$ μm, $\Delta\lambda/\lambda = 0.12\%$, $V = 38$, $n_1 = 1.453$, and $n_2 = 1.438$.

Calculate the pulse spread due to group delay.

Calculate the pulse spread due to material dispersion.

Solution: a. The group delay (or modal delay) is found from

$$\begin{aligned}\Delta\tau_g &= \frac{L(n_1-n_2)}{c}\left(1-\frac{\pi}{V}\right) = \frac{(800)(1.453-1.438)}{3.0\times 10^8}\left(1-\frac{\pi}{38}\right) \\ &= 36.7\times 10^{-9} = 36.7 \text{ ns}\,.\end{aligned} \tag{4.2}$$

b. The value of $\lambda^2 d^2n/d\lambda^2$ at 1,000 nm is estimated from Fig. 3.7b on p. 62 as 0.013. The material delay is found from

$$\begin{aligned}\Delta\tau_m &= -\frac{L}{c}\frac{\Delta\lambda}{\lambda}\lambda^2\frac{d^2n}{d\lambda^2} = -\left(\frac{800}{3.0\times 10^8}\right)(0.0012)(0.013) \\ &= -4.16\times 10^{-11} = -41.6 \text{ ps}\,.\end{aligned} \tag{4.3}$$

3.15. Consider a 9/125 single-mode fiber with $n_1 = 1.48$ and $\Delta = 0.22\%$ as described in the waveguide dispersion example calculation on page 65. Let $L = 1000$ and $\Delta\lambda = 1$ nm.
a.) Plot $\Delta\tau_{wg}$ for wavelengths ranging from 1200 nm to 1400 nm.

b.) Plot $\Delta\tau_{mat}$ for the same range of wavelengths.

c.) Plot the sum of the material pulse spread and the pulse spread due to waveguide dispersion.

Solution: Figure 4.1 shows an Excel spreadsheet for this problem. The columns in the spreadsheet are as follows:

- The first column ("Wavelength") is the wavelength.
- The second column ("V") is the computed value of V from the equation

$$V = \frac{2\pi a n_1\sqrt{2\Delta}}{\lambda}\,. \tag{4.4}$$

- The third column ("Factor #1") is the factor $V d^2(Vb)/dV^2$ as was computed in Prob. 3.14,
- The fourth column ("tau_wg") is the waveguide delay as calculated from the equation,

$$\Delta\tau_{wg} = -\frac{n_2 L\Delta}{c}\frac{\Delta\lambda}{\lambda}\left(V\frac{d^2(Vb)}{dV^2}\right)\,. \tag{4.5}$$

- The fifth column ("Factor #2") is the factor $\lambda^2 d^2n/d\lambda^2$ as calculated in Prob. 3.9 using the Sellmeier equation..
- The sixth column ("tau_m") is the material dispersion delay as computed from the equation

$$\Delta\tau_m = -\frac{L\,\Delta\lambda}{c\lambda}\left(\lambda^2\frac{d^2n}{d\lambda^2}\right)\,. \tag{4.6}$$

Solution to Fiber Optics problem 3.15..
Plot the waveguide dispersion, material dispersion, and the total dispersion for the given fiber.
Factor #1 values computed by Mathview solution to prob. 3.14. (= V d^2(Vb)/dV^2)
Factor #2 values computed by Mathview solution to prob. 3.9. (= λ^2 d^2n/dλ^2)

Wave-length	V	Factor #1	Δτ_wg (ps)	Factor #2	Δτ_mat (ps)	Δτ_total (ps)	λ (nm)
1.20E-06	2.31	0.321	-2.899	0.00284	-7.88	-10.78	1.2
1.21E-06	2.29	0.337	-3.013	0.00244	-6.72	-9.73	1.21
1.22E-06	2.28	0.352	-3.127	0.00204	-5.58	-8.70	1.22
1.23E-06	2.26	0.368	-3.241	0.00165	-4.46	-7.70	1.23
1.24E-06	2.24	0.384	-3.354	0.00126	-3.38	-6.73	1.24
1.25E-06	2.22	0.400	-3.467	0.00087	-2.31	-5.78	1.25
1.26E-06	2.20	0.416	-3.579	0.00048	-1.28	-4.86	1.26
1.27E-06	2.19	0.433	-3.691	0.00010	-0.26	-3.95	1.27
1.28E-06	2.17	0.449	-3.803	-0.00028	0.73	-3.07	1.28
1.29E-06	2.15	0.466	-3.913	-0.00066	1.71	-2.21	1.29
1.30E-06	2.14	0.483	-4.023	-0.00104	2.66	-1.36	1.3
1.31E-06	2.12	0.500	-4.132	-0.00141	3.59	-0.54	1.31
1.32E-06	2.10	0.517	-4.240	-0.00179	4.51	0.27	1.32
1.33E-06	2.09	0.534	-4.347	-0.00216	5.41	1.06	1.33
1.34E-06	2.07	0.551	-4.452	-0.00253	6.29	1.84	1.34
1.35E-06	2.06	0.568	-4.557	-0.00290	7.15	2.60	1.35
1.36E-06	2.04	0.585	-4.661	-0.00326	8.00	3.34	1.36
1.37E-06	2.03	0.603	-4.763	-0.00363	8.84	4.07	1.37
1.38E-06	2.01	0.620	-4.864	-0.00400	9.66	4.79	1.38
1.39E-06	2.00	0.637	-4.963	-0.00436	10.46	5.50	1.39
1.40E-06	1.98	0.654	-5.061	-0.00473	11.26	6.20	1.4

Delta	0.0022	
a	4.50E-06	Parameters from problem
L	1000	
Delta_lam	1.00E-09	
n1	1.480	
n2	1.477	

Figure 4.1: Spreadsheet solution to Problem 3.15.

- The seventh column ("Total") is the sum of the fourth and sixth columns.

The last two columns contain the constant parameters of the problem.

Figure 3.11 is the plot for this problem. The plot shows the fourth, fifth and sixth columns plotted as functions of the first column.

3.16. A fiber has a loss of 0.22 dB/km.

a. Find the effective length of the a long fiber using the long-length approximation.

b. How long must the fiber actually be to have the approximate value of the effective length be within 5% of the actual value?

Solution: We convert the loss coefficient as

$$\alpha_p = 2.3 \times 10^{-3}\alpha = (2.3 \times 10^{-4})(0.22) = 5.06 \times 10^{-5}\ \mathrm{m}^{-1}\,. \tag{4.7}$$

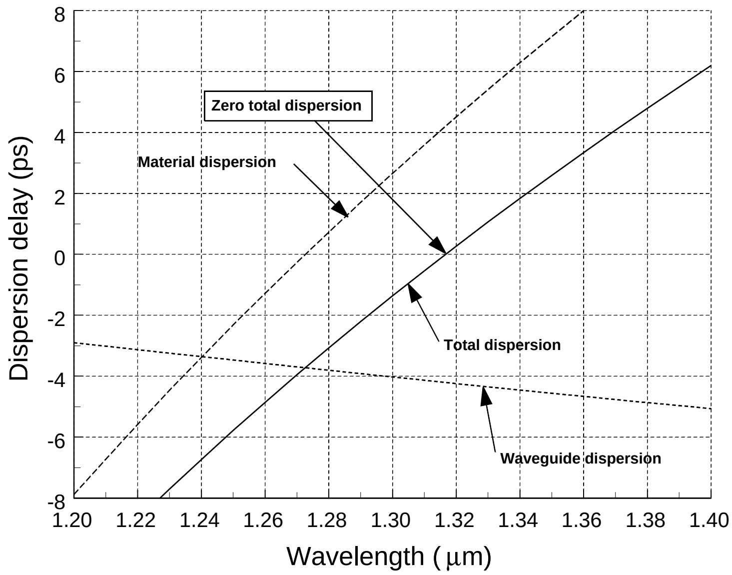

Figure 4.2: Plot of results of Problem 3.15.

a) The value of L_{eff} is found from

$$L_{\text{eff}} = \frac{1 - e^{-\alpha_p L}}{\alpha_p} \approx \frac{1}{\alpha_p} = \frac{1}{5.06 \times 10^{-5}} = 1.976 \times 10^4 \text{ m} = 19.76 \text{ km}\,. \tag{4.8}$$

b) Suppose that we want $L_{\text{eff approx}}$ to be 5% bigger than L_{eff}.

$$\begin{aligned}
L_{\text{eff approx}} &= 1.05 L_{\text{eff}} \\
\frac{1}{\alpha_p} &= 1.05\left(\frac{1 - e^{-\alpha_p L}}{\alpha_p}\right) \\
e^{-\alpha_p L} &= 1 - \frac{1}{1.05} = 0.0476 \\
-\alpha_p L &= \ln(0.0476) = -3.04 \\
L &= \frac{-3.04}{-5.06 \times 10^{-4}} = 6.02 \text{ km}\,.
\end{aligned} \tag{4.9}$$

3.20. Consider a single-mode fiber with an MFD of 10 μm and a loss of 0.2 dB/km at 1550 nm. Find the power limit determined by the Raman scatter.

Solution: The attenuation coefficient is

$$\alpha_p = (2.30 \times 10^{-4})\alpha = (2.30 \times 10^{-3})(0.2) = 4.6 \times 10^{-5} \text{ m}^{-1}\,. \tag{4.10}$$

The effective area is

$$A_{\text{eff}} = \frac{\pi(\text{MFD})^2}{4} = \frac{\pi(10 \times 10^{-6})^2}{4} = 7.85 \times 10^{-11}\ \text{m}^2\,. \tag{4.11}$$

The effective length is

$$L_{\text{eff}} \approx \frac{1}{\alpha_p} = \frac{1}{4.6 \times 10^{-5}} = 21.7\ \text{km}\,. \tag{4.12}$$

The Raman gain at 1550 nm is found as

$$G_R[1550] = G_R[694]\left(\frac{694}{1550}\right)^2 = (0.9 \times 10^{-13})(0.200) = 1.80 \times 10^{-14}\ \text{m} \cdot \text{W}^{-1}\,. \tag{4.13}$$

Finally, we calculate

$$P_R \approx \frac{16A_{\text{eff}}}{G_R[1550]\,L_{\text{eff}}} = \frac{(16)(7.85 \times 10^{-11})}{(1.80 \times 10^{-14})(2.17 \times 10^4)} = 3.22\ \text{W}\,. \tag{4.14}$$

Hence, the Raman threshold is quite high (for a fiber system).

3.21. Calculate the Brillouin scatter limit for the fiber of the previous problem for a bit rate of 1 Gb/s.

Solution: The values of A_{eff} and L_{eff} are the same as in the previous problem. The Brillouin scattering coefficient at 1550 nm is found from

$$\begin{aligned} G_B[1550] &= G_B[1000]\left(\frac{1000}{1550}\right)^2 = \left(4.59 \times 10^{-11}\right)(0.416) \\ &= 1.87 \times 10^{-11}\ \text{m/W}\,. \end{aligned} \tag{4.15}$$

We then calculate

$$P_B \approx \frac{21A_{\text{eff}}}{G_B[1550]\,L_{\text{eff}}} = \frac{(21)(7.85 \times 10^{-11})}{(1.87 \times 10^{-11})(2.17 \times 10^3)} = 40.5\ \text{mW}\,. \tag{4.16}$$

We have no information about the source linewidth (i.e., $\Delta\nu_{\text{pump}}$), so we will assume that Brillouin linewidth is larger than the source linewidth. This implies that no correction factor due to the linewidth is required. Hence, the Brillouin threshold is much lower than the Raman threshold.

3.23. We want to compute the fraction of a tensile load that is carried by the coating of a fiber. The fraction of the total stress σ_{total} that is carried by the coating is found from

$$\frac{\sigma_{\text{coating}}}{\sigma_{\text{total}}} = \frac{E_{\text{coating}}A_{\text{coating}}}{E_{\text{coating}}A_{\text{coating}} + E_{\text{fiber}}A_{\text{fiber}}} \tag{4.17}$$

where E_{coating} and E_{fiber} are the Young's modulus of the coating and fiber material, respectively, and A_{coating} and A_{fiber} are the cross-section areas of the coating and fiber, respectively. Consider a 62.5/125 fiber that has a polymer coating that is 0.05 mm thick surrounding it. The Young's modulus of the coating polymer is 350 MPa and of glass is 71.9 GPa.

a.) Calculate the fraction of the total applied stress that is carried by the coating.

b.) Calculate the fraction of the total applied stress that is carried by the fiber.

c.) Suppose that the fiber were double-coated with two different concentric coatings. What do you think would be the expression for the fraction of the total stress carried by the interior coating?

Solution: We are given that $E_{\text{coating}} = 3.5 \times 10^8$ and $E_{\text{fiber}} = 7.19 \times 10^{10}$. The diameter of the cable (including the 50 μm thick coating) is 225 μm. The areas of the fiber and the coating are

$$A_{\text{fiber}} = \frac{\pi d_{\text{fiber}}^2}{4} = \frac{\pi(125 \times 10^{-6})^2}{4} = 1.227 \times 10^{-8} \text{ m}^2 \tag{4.18}$$

and

$$\begin{aligned} A_{\text{coating}} &= \frac{\pi d_{\text{cable}}^2}{4} - \frac{\pi d_{\text{fiber}}^2}{4} = \frac{\pi(225 \times 10^{-6})^2}{4} - \frac{\pi(125 \times 10^{-6})^2}{4} \\ &= 2.75 \times 10^{-8} \text{ m}^2 . \end{aligned} \tag{4.19}$$

a.) The fraction of the stress carried by the coating is

$$\begin{aligned} \frac{\sigma_{\text{coating}}}{\sigma_{\text{total}}} &= \frac{E_{\text{coating}} A_{\text{coating}}}{E_{\text{coating}} A_{\text{coating}} + E_{\text{fiber}} A_{\text{fiber}}} \\ &= \frac{(3.5 \times 10^8)(2.75 \times 10^{-8})}{(3.5 \times 10^8)(2.75 \times 10^{-8}) + (7.19 \times 10^{10})(1.227 \times 10^{-8})} \\ &= 0.01079 = 1.079\% . \end{aligned} \tag{4.20}$$

b.) The fraction of the stress carried by the fiber is

$$\begin{aligned} \frac{\sigma_{\text{fiber}}}{\sigma_{\text{total}}} &= \frac{E_{\text{fiber}} A_{\text{fiber}}}{E_{\text{coating}} A_{\text{coating}} + E_{\text{fiber}} A_{\text{fiber}}} \\ &= \frac{(7.19 \times 10^{10})(1.227 \times 10^{-8})}{(3.5 \times 10^8)(2.75 \times 10^{-8}) + (7.19 \times 10^{10})(1.227 \times 10^{-8})} \\ &= 0.989 = 98.9\% . \end{aligned} \tag{4.21}$$

An alternative solution is to recognize that

$$\frac{\sigma_{\text{fiber}}}{\sigma_{\text{total}}} = 1 - \frac{\sigma_{\text{coating}}}{\sigma_{\text{total}}} . \tag{4.22}$$

c.) By analogy with the given equation we would expect

$$\frac{\sigma_{\text{inner coat}}}{\sigma_{\text{total}}} = \frac{E_{\text{inner coat}} A_{\text{inner coat}}}{E_{\text{inner coat}} A_{\text{inner coat}} + E_{\text{outer coat}} A_{\text{outer coat}} + E_{\text{fiber}} A_{\text{fiber}}} . \tag{4.23}$$